TITLE: Application of 2 8-4 Fractional Factorials in Screening of Variable Affecting the Performance of Dry Process Zinc Battery Electrodes

APPLICATION OF 2^{8-4} FRACTIONAL FACTORIALS IN SCREENING OF VARIABLES AFFECTING THE PERFORMANCE OF DRY PROCESS ZINC BATTERY ELECTRODES

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Among its research and development activities on zinc-silver oxide batteries for special applications, USAELRDL is investigating the preparation of zinc electrodes by dry processes. These involve the application of dendritic zinc powders under pressure to the grids. The interlocking properties of the dendritic zinc particles make it possible to form the electrodes with moderate pressures such that the porosity and related high surface area of the electrode is not destroyed. It is expected that dry process zinc electrodes will have many advantages over conventional electrodeposited sponge zinc electrodes, including higher discharge efficiency, greater uniformity of performance and better adaptability to mechanized production with resultant economics.

Due to the large number of variables affecting the discharge performance of the electrodes, it was decided to design and conduct a fractional factorial experiment to isolate the significant variables. These variables, and any controlling interactions between them, could then be studied further to arrive at the optimum conditions for the production of electrodes of maximum discharge efficiency. The fractional factorial experiment was thus intended for the preliminary screening of all major variables acting simultaneously. As such, it was recognized that it is the most efficient and economical process known for accomplishing this, in addition to providing valuable data on interactions between the variables which cannot be obtained from the more widely used one or two at a time variable investigations.

There were two categories of variables in the electrode investigation, those related to the electrolytic formation of the dendritic zinc powders and those related to the electrode preparation itself. Although several more variables were considered, it was decided to limit the number of variables to eight, keeping all other factors constant. The eight selected variables are shown in Figure 1. High and low levels as shown were assigned to each variable. Although considerable thought was given to determination of the levels, it is seen in retrospect that wider ranges might have been assigned in some instances. Based on available literature, these were, however, considered sufficiently wide ranges.

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*Figures can be found at the end of this article.

Variable G, the electrolyte temperature during the plating operation, turned out to be impossible to control with the plating equipment which was prepared for the experiment and with the selected plating current densities. The experiment thus became one involving seven variables, each at two levels. Rather than to eliminate G, however, it was felt that this paper would serve a broader purpose if it gave the fractional factorial procedures for the study of eight as well as seven variables. Variable G should, therefore, be considered both in and out of the figures and analysis.

Having established the variables and the high and low levels, the fractional replicate design was established, as shown in Figure 2. This is the first design, a second having been run later for reasons to be discussed. The design involves eight variables (or seven) each at two levels to be studied with a total of sixteen different electrodes. A full 2^8 factorial experiment, eight variables each at two levels, would involve 2^8 or 256 trials (128 trials for a 2^7 factorial). Therefore, this design represents a one sixteenth fraction of the 2^8 factorial (one eighth of the 2^7), expressed as a 2^{8-4} design (or 27-3 for seven factors). The design is based on extension of a basic 2^4 or 16 trials. Thus the first four variables are arranged in standard order for the 2^4 factorial. The other variables are then introduced by making a basic assumption that three factor interactions between the first four variables are negligible. Therefore E is introduced by equating it to the interaction between variables A, B, and C. Regarding the high level as plus and the low level as minus, the level of E for the first box is $- \times - \times - = -$, $+ \times - \times - = +$ for the second box, etc. Similarly, variable F is introduced to equating it to the BCD interaction, variable G to the ABD interaction and variable H to the ACD interaction. Normally in a seven variable design, the seventh variable would be equated to ABD instead of ACD as was done here since variable G was dropped out. However, this does not affect the 2^{7-3} experiment in any way.

Having thus established the fractional design, the sixteen electrodes were prepared in accordance with the high and low level criteria for each variable. The electrodes were then tested one at a time under standard and carefully controlled conditions to give the sixteen yields or responses as shown. The response is the discharge efficiency of the electrode expressed in percent as the ratio of the output capacity (at a discharge current density of 1.1 amp/square inch to a 0.3 volt change for the electrode) to the theoretical capacity of the zinc active material. For comparison the conventional sponge zinc electrodes give an average efficiency of about 20% under comparable discharge conditions. The average of the sixteen responses is 31.2%. Before proceeding with the analysis of the responses, it should be noted how the design can be used to relate the sixteen mean effects obtained from the response analysis to the variables and their two factor interactions. Consider the high level boxes in the first four columns. The first mean effect, no high levels, will be for twice the average. The second will be for variable A, the third for B, the fourth for the AB interaction, the fifth for C and so on. The eighth (A, B and C at the high level) will be for variable E which was originally equated to ABC. Similarly the twelfth for G, the fourteenth for H and the fifteenth for F. Since this is a one sixteenth fraction, each of these principal effects will be confused with fifteen other effects. However, these for each of the eight variables will be three factor or higher order interactions which are considered negligible, the basis on which the design was established. Each two factor interaction, AB for example, will be confused with fifteen other effects of which three are other two factor interactions. The sixteenth row of the First Design will represent such a combination of four 2 factor interactions.

The analysis of the sixteen responses is shown in Figure 3. The technique used here is the Yates' Algorithm which is a rapid method for obtaining the same mean effects that would be obtained from a formal and lengthy analysis of variance. The Yates' Algorithm is applicable to any factorial experiment. Its advantages become more apparent the larger the experiment.

The mechanics of the Yates' calculations are very simple. The first figure in Column (1) is the sum of responses 1 and 2; the second, the sum of responses 3 and 4, etc. The ninth figure is the sum of responses 1 and 2 with the sign of response 1 reversed. Column (2) is derived from Column (1) in the same manner. Additional columns are introduced until a column is completed the first figure of which is equal to the sum of the responses. The arithmetic in each column is checked before proceeding to the next column. The sum of Column (1) is equal to twice the sum of the even numbered responses; the sum of Column (2) is equal to four times the sum of every second even numbered response; the sum of Column (3) is equal to eight times the sum of every fourth even numbered response; and the sum of Column (4) is equal to sixteen times the sixteenth response. The mean effects are obtained by dividing each figure of Column (4) by eight. The sum of the mean effects is also checked. The 62.363 figure, twice the average, is not used in the subsequent analysis. The effects measured by the mean effects are given in the last column. The effects A, B, AB, C, AC, etc. are those as previously read off the first design chart. The three factor and higher order interactions which they are confused with are of no importance since the design is based on their being assumed negligible. The other two factor interactions are important and must be known. They are obtained from the effects A, B, AB, etc. and the defining contrasts of the design. These defining contrasts are obtained from the equating that had been done in establishing the fractional design; E = ABC, F = BCD, G = ABD and H = ACD. Sixteen defining contrasts are required for the eight variable design. The first defining contrast is always I, the next four are ABCE, BCDF, ABDG and ACDH. The remaining eleven are found by exhaustively multiplying these contrasts using the rule that like factors are cancelled out.

(1) I
(2) $ABC \times E = ABCE$
(3) $BCD \times F = BCDF$
(4) $ABD \times G = ABDG$
(5) $ACD \times H = ACDH$
(6) $A\cancel{B}\cancel{C}E \times \cancel{B}\cancel{C}DF = ADEF$
(7) $\cancel{A}\cancel{B}CE \times \cancel{A}\cancel{B}DG = CDEG$
(8) $\cancel{A}B\cancel{C}E \times \cancel{A}\cancel{C}DH = BDEH$
(9) $\cancel{B}C\cancel{D}F \times A\cancel{B}\cancel{D}G = ACFG$
(10) $B\cancel{C}\cancel{D}F \times A\cancel{C}\cancel{D}H = ABFH$
(11) $\cancel{A}B\cancel{D}G \times \cancel{A}C\cancel{D}H = BCGH$
(12) $\cancel{A}\cancel{B}\cancel{C}E \times \cancel{B}\cancel{C}\cancel{D}F \times \cancel{A}B\cancel{D}G = BEFG$
(13) $\cancel{A}\cancel{B}\cancel{C}E \times \cancel{B}\cancel{C}\cancel{D}F \times \cancel{A}C\cancel{D}H = CEFH$
(14) $\cancel{A}\cancel{B}\cancel{C}E \times \cancel{A}\cancel{B}\cancel{D}G \times A\cancel{C}\cancel{D}H = AEGH$
(15) $\cancel{B}\cancel{C}\cancel{D}F \times \cancel{A}\cancel{B}\cancel{D}G \times \cancel{A}\cancel{C}DH = DFGH$
(16) $\cancel{A}\cancel{B}\cancel{C}E \times \cancel{B}\cancel{C}\cancel{D}F \times \cancel{A}B\cancel{D}G \times ACDH = ABCDEFGH$

(These sixteen defining contrasts may also be obtained by simply reading off the low level boxes for each of the sixteen trials in the First Design chart. However this procedure will not apply in all cases, e.g. in the 2^{7-3} it would give sixteen defining contrasts when only eight are required.)

The principal effects are found by multiplying the first effect, A, B, AB, etc., by the sixteen defining contrasts. The two factor interactions confused with AB are found for example to be CE, DG and FH.

$$AB \times I = AB$$
$$\cancel{A}\cancel{B} \times \cancel{A}\cancel{B}CE = CE$$
$$\cancel{A}\cancel{B} \times \cancel{A}\cancel{B}DG = DG$$
$$\cancel{A}\cancel{B} \times \cancel{A}\cancel{B}FH = FH$$

The other twelve effects confused with AB, all three factor or higher order interactions, could be found with the other twelve defining contrasts if desired. The total 256 effects found by multiplying each starting effect by the sixteen defining contrasts would give the full 256 effects, from I to the interaction ABCDEFGH, which would be obtained in conducting a full 2^8 factorial experiment.

The defining contrasts for the 2^{7-3} experiment are I, ABCE, BCDF, ACDH, AEDF, BDEH, ABFH, and CEFH. The effects measured are found in the same way as for the eight variable design. The only difference is that, in this case, all effects involving G drop out. The twelfth set of effects measured, identified by the asterisk, then becomes eight interactions, all three factor or higher order.

The fifteen mean effects with their identifying effects are then ordered by arranging them in order of magnitude without regard to sign. The thus ordered set of mean effects is then arranged in a half-normal plot as shown in Figure 4 to interpret the relative significance of the effects. The ordinate is the order number of the fifteen effects from smallest to largest. The abscissa gives the mean effect magnitudes. The fifteen points which are plotted are identified by the proper major effects and two factor interactions. The plot is given for both the full eight variables, and the actual seven variable experiment. In the latter case, the G factor and the G interactions drop out. The asterisk again denotes high order interactions. In the plot an error best straight line has been drawn through the lowest seven points. High magnitude effects falling significantly off the line are judged to be distinct from error and therefore controlling factors in the process being investigated. In a plot of this type, it is considered unusual, however, to have a well defined error line with as many as eight points falling clearly off it. To gain insight into this unusual behavior as well as to gain more precision in the estimation of all of the effects, it was decided to conduct a second phase of the experiment, another group of sixteen electrodes differing from the first. Since an interaction between A (the highest magnitude effect) and B appeared reasonable, and since F and H were both high magnitude effects which might interact with each other, it was decided to establish the second design in such a way as to separate the AB and FH interactions. Although this can be done in several equally effective ways, it was decided to do it by reversing the levels for factors A and D, giving the design as shown in Figure 5.

This design is identical to the first except for the reversal of levels of variables A and D. Sixteen electrodes were prepared in accordance with the indicated variable levels. The electrodes were then tested to give the listed responses. The next step in the experiment was to determine the mean effects generated by these responses and to combine them with the mean effects resulting from the first design. This may be done in two ways. The first is to combine the two sets of sixteen responses into an overall group of thirty-two and then to conduct a Yates' computation to arrive at the thirty-two mean effects. The second way is to conduct a Yates' computation on the second design responses and then combine the mean effects with those of the first design. The first method is less time consuming and therefore preferable. The second method gives a clearer picture of the separation of effects and is therefore now given. (The first method is given in Appendix A-1)

Figure 6 shows the Yates' computations on the responses from the second design. The operations are identical to those as described for the first design. It is noted that the signs of all elements involving A and D in the Effects Measured column are now minus since the levels of A and D had been reversed. This reversal of signs permits the separation of effects as shown in Figure 7.

The mean effects derived from the first design are given in the column marked X. Those derived from the second design are given in the column marked Y. An example of the computation to separate the effects is as follows:

The second mean effect in the X column is for variable A. The similar mean effect in the Y column is for minus variable A with the difference in the absolute magnitudes of the mean effects being due to experimental error. Reversing the sign of the column Y mean effect and averaging it with that of the column X mean effect will give the -4.81 mean effect for variable A and certain high order interactions confused with it. 1/2(X+Y), -1.36, then gives the mean effect for the remaining high order interactions originally confused with A. Similar calculations are performed to separate all of the other effects, those containing A or D from each of the others. The thirty-one statistics thus obtained, not including the 61.22 (twice the average) figure, are then arranged in order of magnitude without respect to sign and plotted in a thirty-one factor half-normal plot as shown in Figure 8.

It is seen that the error best straight line is now established by twenty-three of the thirty-one points reflecting the greater precision achieved by doubling the experiment and thereby reducing the variance of the estimated effects by one-half. Of the eight points which are clearly off the line, two of them, denoted by asterisks, are combinations of high order interactions. Their relative significance cannot be interpreted within the limits of the experiment. As will be seen, equating them to zero will not affect the results. The BF interaction is not far off the line and its significance may be questioned. Variables E and A are both clearly controlling factors in the efficiency of the zinc electrodes. The signs of their mean effects are both minus, indicating that higher efficiencies can be obtained at the lower levels of the ranges studied, in other words at the lower pressing temperature, 80° F, and with the smaller weight of zinc per plate. The interpretation of variables H, the formation current density, and F, the presence or absence of zinc oxide in the formation electrolyte, is more complex. This results from the probable significance of FH, the interaction between them (CE is very unlikely to be significant due to the low magnitude of C). In general when an interaction is large, as in this case, the corresponding mean effects cease to have much meaning. The effect of F is clearly dependent upon the level of H and vice versa. The three effects F, H and their interaction FH may best be interpreted as a single highly significant effect. Further experimental work at intermediate levels for the two variables is definitely indicated.

The final step in the analysis was to determine if the conclusion was correct that only the effects E, A and the combined effects of H, F and FH were significant, i.e., distinct from experimental error. Part of the purpose of this final step was to determine if the entire experiment was valid, in other words, that there were no large errors made in the actual responses which could have seriously altered the mean effects. The procedure used was to determine the standard error of the individual observed responses by analyzing the thirty-one mean effects. A second standard error, for the differences between observed and predicted responses, was then obtained with the predicted responses based on the assumption that all mean effects other than those for E, A, H, FH and F were indeed zero. If the two standard errors would then be equivalent, both the total experiment and the conclusions derived from it would be proved valid.

Since the error straight line on the final half-normal plot was established by the twenty-three lowest magnitude points, a half-normal plot for these points was prepared. The standard error for the individual observed responses as derived from this plot was 3.0. (See A-5). All mean effects other than the twice the average effect, E, A, H, FH and F were then equated to zero and a reverse Yates' computation was conducted to obtain the thirty-two predicated responses (See A-2). These responses were compared with the observed responses and a list of the thirty-two differences between the observed and predicted responses was prepared (See A-3). The magnitudes of the differences were plotted on normal probability paper. A standard error was obtained from this plot for the difference between individual observed responses and individual predicted responses. The value of this standard error was 3.2 (See A-4). This was in excellent agreement with the standard error of the individual observed responses, thus proving the validity of the experiment and the conclusions derived from it.

In conclusion, a fractional factorial designed experiment, involving two 2^{7-3} fractional designs, has been conducted to determine the significance of seven major variables, and two factor interactions between them, on the discharge efficiency of the dry process zinc battery electrodes. A total of thirty-two electrodes was prepared and tested. Analysis of their responses has indicated the controlling influence of two of the variables, pressing temperature and the amount of zinc per plate, and of the interaction between two other variables relating to the plating conditions under which the zinc material was prepared. The other three variables have been shown to be unimportant in comparison, within the range of levels selected. The experiment has fulfilled its basic purpose, narrowing down the range of variables to permit extensive investigation of the truly important variables in order to arrive at the optimum electrode preparation procedures in the most expeditious manner.

Figure 1

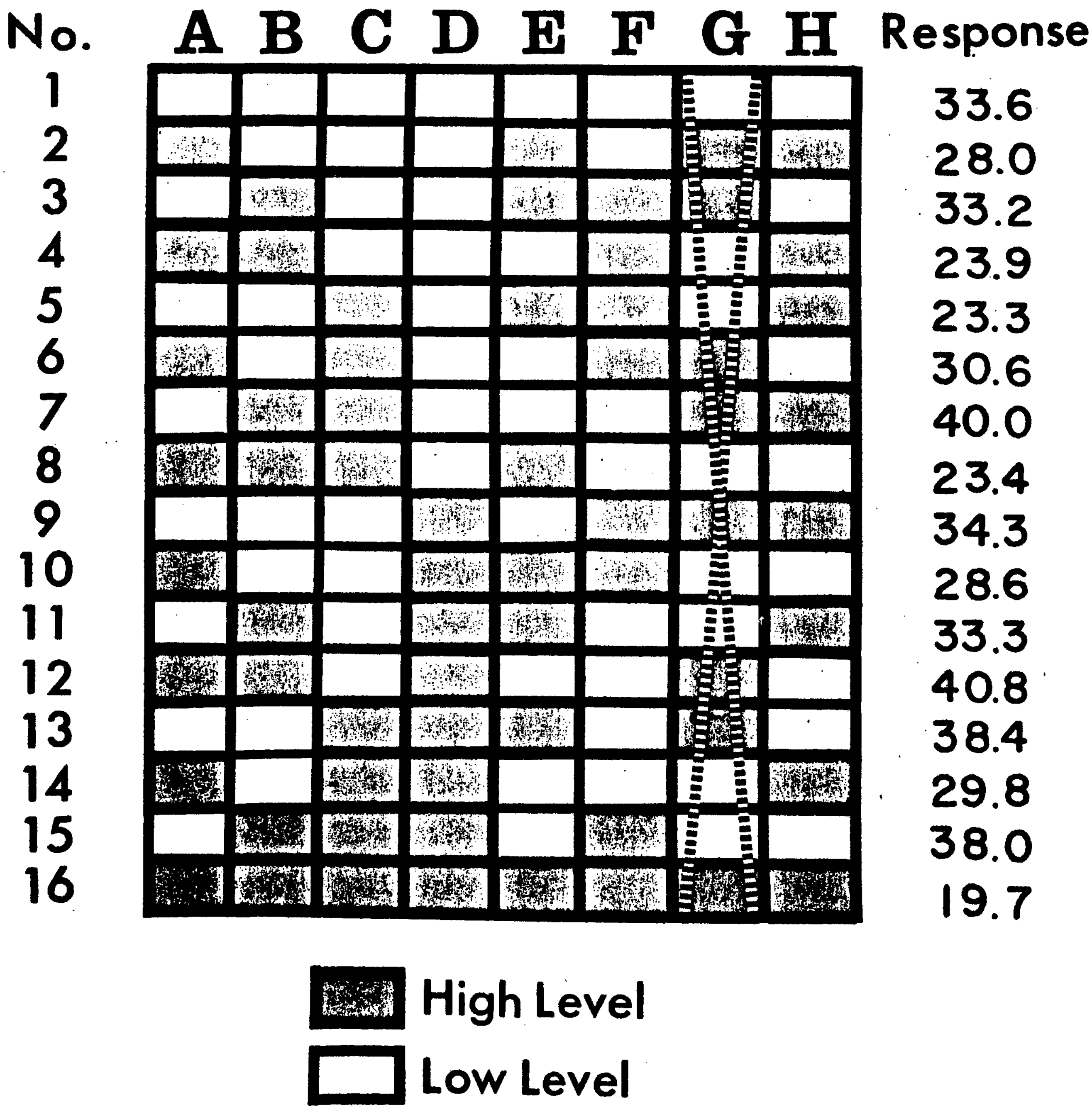
Experiment Variables

	Pressing Variables	Units	High	Low
A	Zinc weight	grams	5.23	2.62
B	Pressure	psi	1,840	1,230
C	Particle size	sieve mesh	100	200
D	Pressure time	minutes	15	1
E	Pressure temp.	°F	300	80
	Formation Variables			
F	ZnO in electrolyte	gm./liter	20	0
~~G~~	~~Electrolyte temp.~~	~~°F~~	~~100~~	~~80~~
H	Current density	amp./sq. in.	1.0	0.75

Figure 2

FIRST DESIGN

Variables



FIRST DESIGN ANALYSIS, YATES' ALGORITHM

NO.	RESPONSE	(1)	(2)	(3)	(4)	MEAN EFFECTS (4)/8	EFFECTS MEASURED
1	33.6	61.6	118.7	236.0	498.9	62.363	-
2	28.0	57.1	117.3	262.9	-49.3	-6.163	A
3	33.2	53.9	137.0	-24.2	5.7	0.713	B
4	23.9	63.4	125.9	-25.1	-24.1	-3.013	AB + CE + ~~DG~~ + FH
5	23.3	62.9	-14.9	5.0	-12.5	-1.563	C
6	30.6	74.1	-9.3	0.7	-23.1	-2.888	AC + BE + DH + ~~FG~~
7	40.0	68.2	1.8	-27.6	-7.7	-0.963	AE + BC + DF + ~~GH~~
8	23.4	57.7	-26.9	3.5	-43.1	-5.388	E
9	34.3	-5.6	-4.5	-1.4	26.9	3.363	D
10	28.6	-9.3	9.5	-11.1	-0.9	-0.113	AD + ~~BG~~ + CH + EF
11	33.3	7.3	11.2	5.6	-4.3	-0.538	~~AG~~ + BD + CF + EH
12	40.8	-16.6	-10.5	-28.7	31.1	3.888	~~G~~*
13	38.4	-5.7	-3.7	14.0	-9.7	-1.213	AH + BF + CD + ~~EG~~
14	29.8	7.5	-23.9	-21.7	-34.3	-4.288	H
15	38.0	-8.6	13.2	-20.2	-35.7	-4.463	F
16	19.7	-18.3	-9.7	-22.9	-2.7	-0.338	AF + BH + ~~CG~~ + DE
CHECKS	498.9	449.6	431.2	344.8	315.2	39.396	

449.6
431.2
344.8
315.2
39.400

*3 factor and higher order interactions

Figure 3

Figure 4

Half-Normal Plot, First Design

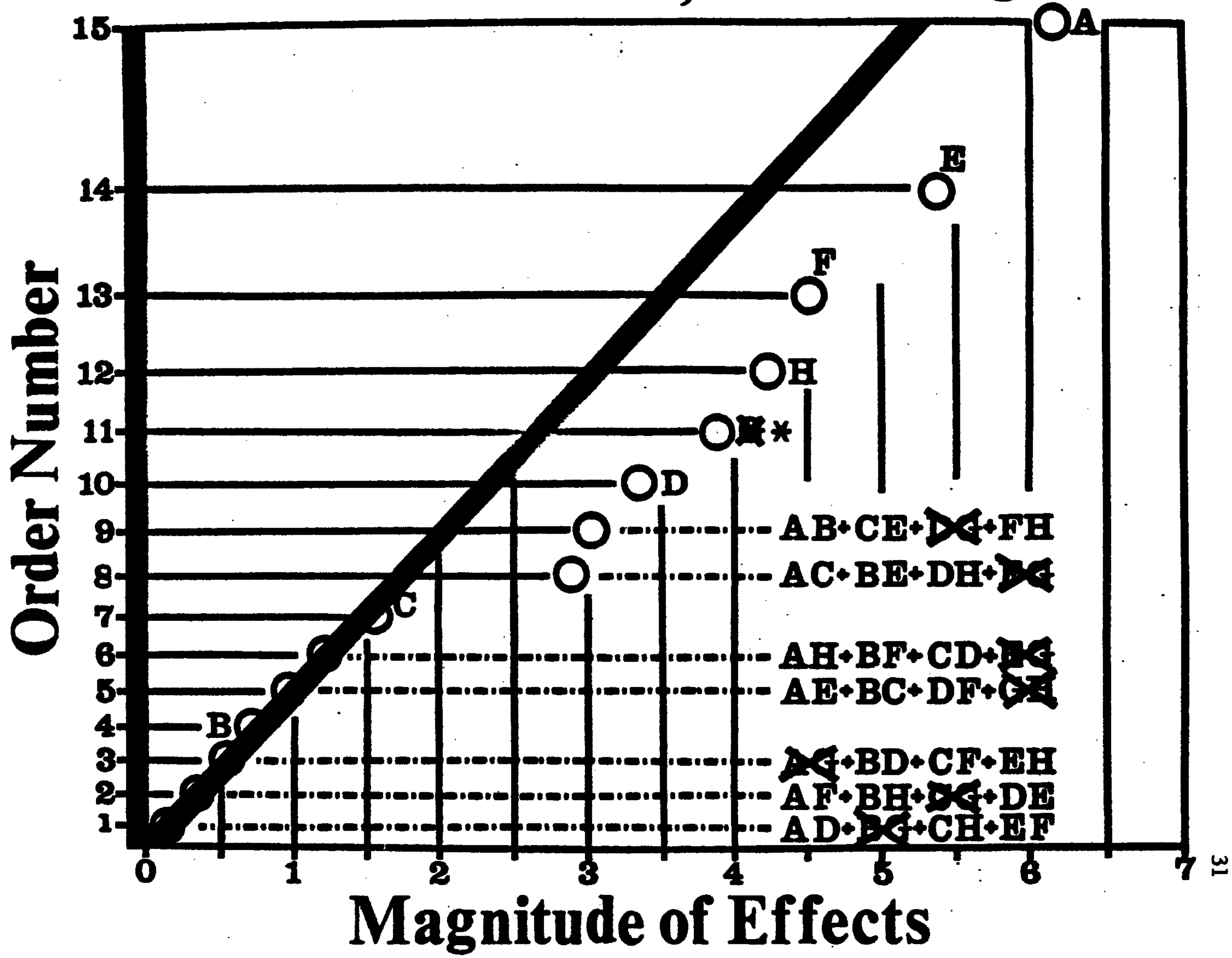
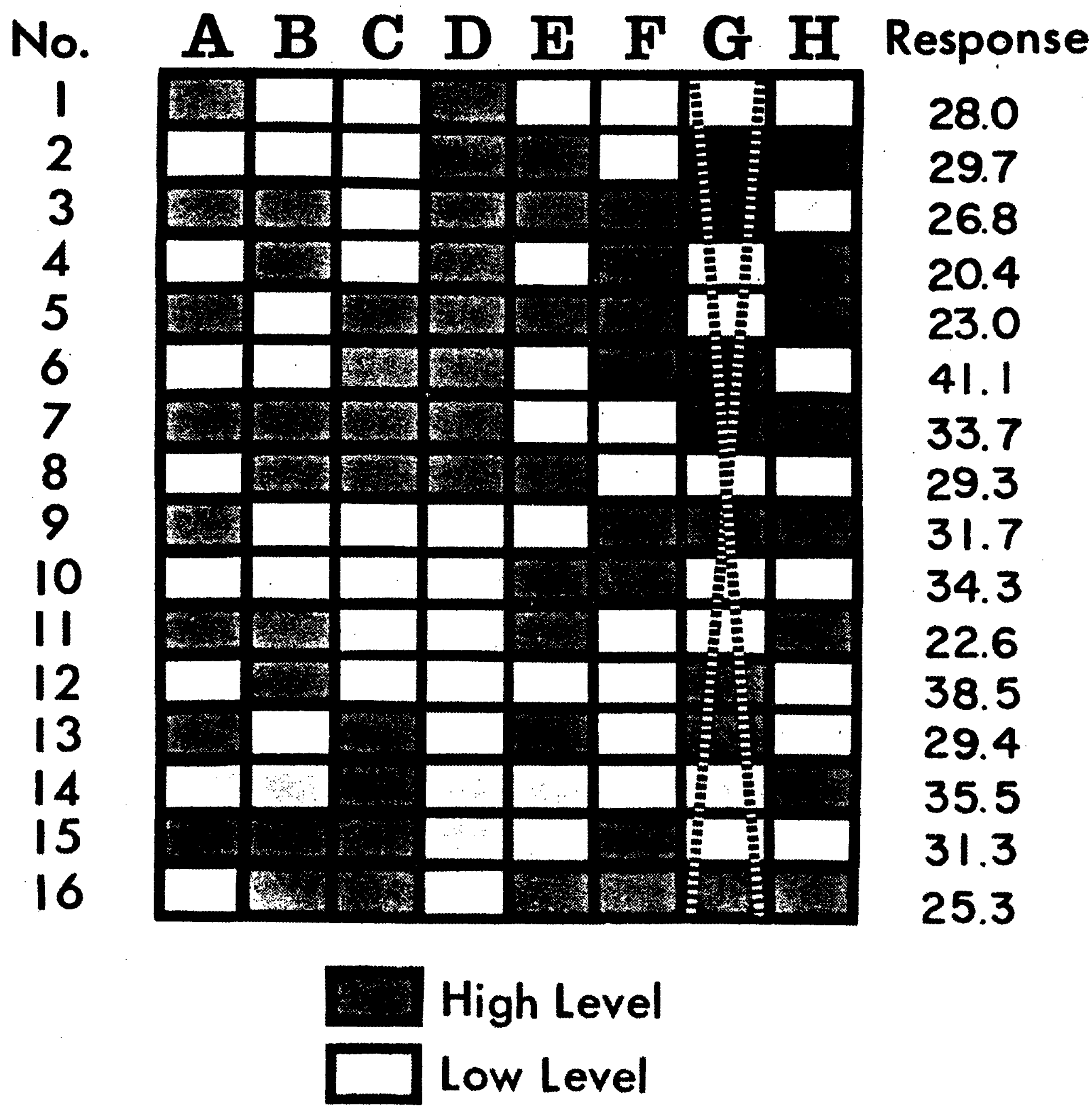


Figure 5

SECOND DESIGN

Variables



SECOND DESIGN ANALYSIS, YATES' ALGORITHM

NO.	RESPONSE	(1)	(2)	(3)	(4)	MEAN EFFECTS (4)/8	EFFECTS MEASURED
1	28.0	57.7	104.9	232.0	480.6	60.075	-
2	29.7	47.2	127.1	248.6	27.6	3.450	-A
3	26.8	64.1	127.1	9.0	-24.8	-3.100	B
4	20.4	63.0	121.5	18.6	-29.4	-3.675	-AB + CE - ~~DG~~ + FH
5	23.0	66.0	-4.7	-11.6	16.6	2.075	C
6	41.1	61.1	13.7	-13.2	0.0	0.000	-AC + BE - DH + ~~FG~~
7	33.7	64.9	18.5	-30.6	6.0	0.750	-AE + BC - DF + ~~GH~~
8	29.3	56.6	0.1	1.2	-39.8	-4.975	E
9	31.7	1.7	-10.5	22.2	16.6	2.075	-D
10	34.3	-6.4	-1.1	-5.6	9.6	1.200	AD + ~~BG~~ + CH + EF
11	22.6	18.1	-4.9	18.4	-1.6	-0.200	-~~AG~~ - BD + CF + EH
12	38.5	-4.4	-8.3	-18.4	31.8	3.975	~~G~~*
13	29.4	2.6	-8.1	9.4	-27.8	-3.475	-AH + BF - CD + ~~EG~~
14	35.5	15.9	-22.5	-3.4	-36.8	-4.600	H
15	31.3	6.1	13.3	-14.4	-12.8	-1.600	F
16	25.3	-6.0	-12.1	-25.4	-11.0	-1.375	-AF + BH + ~~CG~~ - DE
CHECKS	480.6	508.2	454.0	436.8	404.8	50.600	

508.2
454.0
436.8
404.8
50.600

*3 factor and higher order interactions

Figure 6

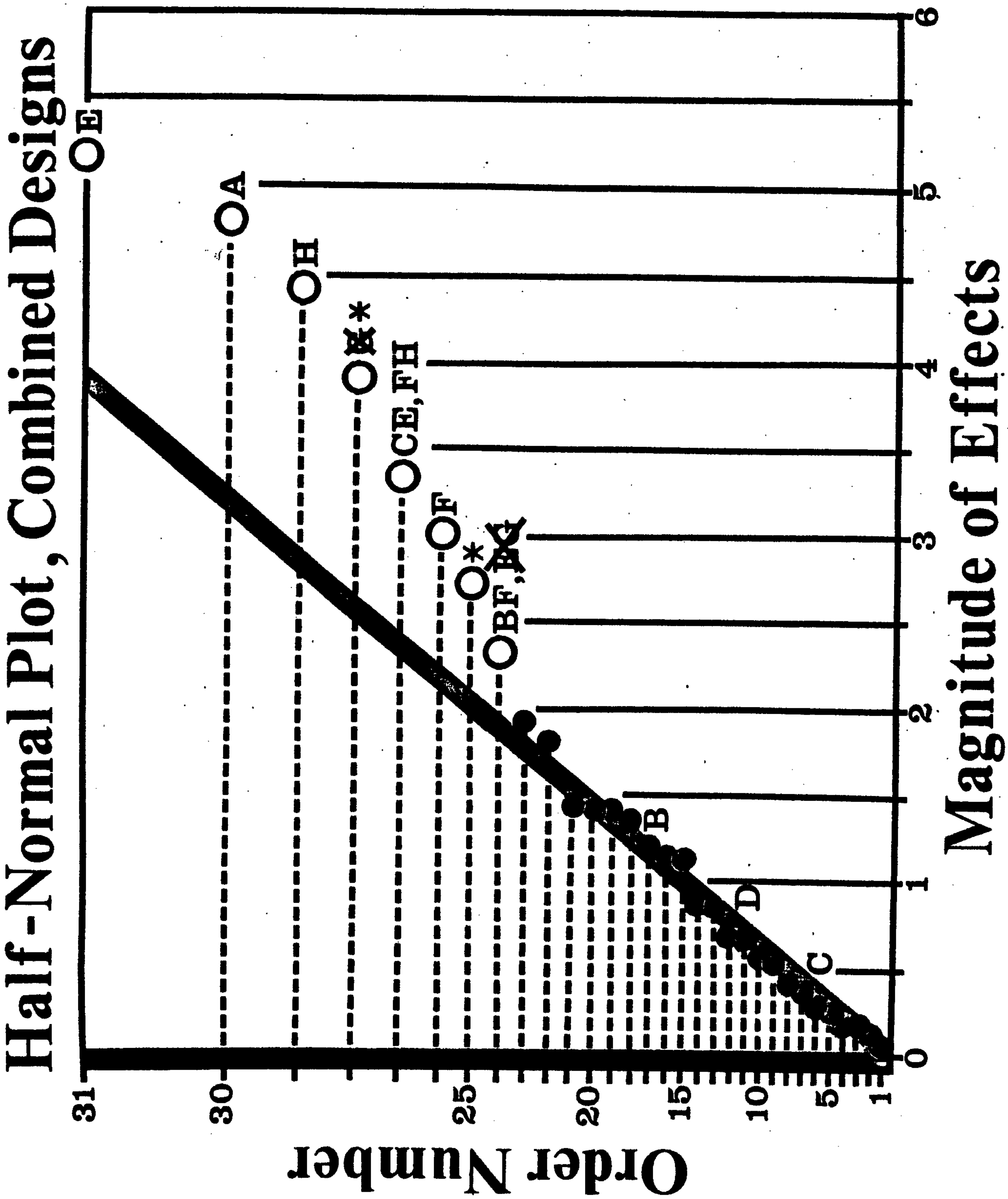
DETERMINATION OF EFFECTS, COMBINED DESIGNS

No.	X Mean Effects First Design	Y Mean Effects Second Design	½(X + Y)	Effects Measured	½(X - Y)	Effects Measured
1	62.363	60.075	61.22	-	1.14	Blocks
2	-6.163	3.450	-1.36	*	-4.81	A
3	0.713	-3.100	-1.19	B	1.91	*
4	-3.013	-3.675	-3.34	CE + FH	0.33	AB + ~~DG~~
5	-1.563	2.075	0.26	C	-1.82	*
6	-2.888	0.000	-1.44	BE + ~~EG~~	-1.44	AC + DH
7	-0.963	0.750	-0.11	BC + ~~GH~~	-0.86	AE + DF
8	-5.388	-4.975	-5.18	E	-0.21	*
9	3.363	2.075	2.72	*	0.64	D
10	-0.113	1.200	0.54	AD + ~~BG~~ + CH + EF	-0.66	*
11	-0.538	-0.200	-0.37	CF + EH	-0.17	~~AG~~ + BD
12	3.888	3.975	3.93	~~G~~*	-0.04	*
13	-1.213	-3.475	-2.34	BF + ~~EG~~	1.13	AH + CD
14	-4.288	-4.600	-4.44	H	0.16	*
15	-4.463	-1.600	-3.03	F	-1.43	*
16	-0.338	-1.375	-0.86	BH + ~~CG~~	0.52	AF + DE
			45.01		-5.61	
			-5.61			
			34.90			
			x 8			
			315.20 (Check)			

* 3 factor and higher order interactions

Figure 7

Figure 8



Half-Normal Plot, Combined Designs
Magnitude of Effects
Order Number
E
A
H
CE,FH
F
B
D
C
0
1
2
3
4
5
6
1
5
10
15
20
25
30
31

APPENDIX

This appendix contains supplemental data as follows:

A-1. Yates' Algorithm computation for thirty-two responses, combination of Designs 1 and 2.

A-2. Reverse Yates' Algorithm computation for thirty-two effects, assuming all effects are zero except those for average, E, A, H, FH and F.

A-3. Comparison of observed and predicted responses.

A-4. Probability plot to obtain standard error of individual differences between observed and predicted responses.

A-5. Half-normal plot of twenty-three mean effects to obtain standard error of individual observed responses.

A-1. Yates' Algorithm computation for thirty-two responses, combination of Designs 1 and 2.

No.	Responses	(1)	(2)	(3)	(4)	(5)	(5)/16	Effects Measured
1	33.6	61.6	118.7	236.0	498.9	979.5	61.22	-
2	28.0	57.1	117.3	262.9	480.6	-21.7	-1.36	*
3	33.2	53.9	137.0	232.0	-49.3	-19.1	-1.19	B
4	23.9	63.4	125.9	248.6	27.6	-53.5	-3.34	CE+FH
5	23.3	62.9	104.9	-24.2	5.7	4.1	0.26	C
6	30.6	74.1	127.1	-25.1	-24.8	-23.1	-1.44	BE+FG
7	40.0	68.2	127.1	9.0	-24.1	-1.7	-0.11	BC+GH
8	23.4	57.7	121.5	18.6	-29.4	-82.9	-5.18	E
9	34.3	57.7	-14.9	5.0	-12.5	43.5	2.72	*
10	28.6	47.2	-9.3	0.7	16.6	8.7	0.54	AD+BG+CH+EF
11	33.3	64.1	1.8	-11.6	-23.1	-5.9	-0.37	CF+EH
12	40.8	63.0	-26.9	-13.2	0	62.9	3.93	G
13	38.4	66.0	-4.7	-27.6	-7.7	-37.5	-2.34	BF+EG
14	29.8	61.1	13.7	3.5	6.0	-71.1	-4.44	H
15	38.0	64.9	18.5	-30.6	-43.1	-48.5	-3.03	F
16	19.7	56.5	0.1	1.2	-39.8	-13.7	-0.86	BH+CG
17	28.0	-5.6	-4.5	-1.4	26.9	-18.3	-1.14	-Blocks
18	29.7	-9.3	9.5	-11.1	16.6	76.9	4.81	-A
19	26.8	7.3	11.2	22.2	-0.9	-30.5	-1.91	-*
20	20.4	-16.6	-10.5	-5.6	9.6	-5.3	-0.33	-AB-DG
21	23.0	-5.7	-10.5	5.6	-4.3	29.1	1.82	-*
22	41.1	7.5	-1.1	-28.7	-1.6	23.1	1.44	-AC-DH
23	33.7	-8.6	-4.9	18.4	31.1	13.7	0.86	-AE-DF
24	29.3	-18.3	-8.3	-18.4	31.8	3.3	0.21	-*
25	31.7	1.7	-3.7	14.0	-9.7	-10.3	-0.64	-D
26	34.3	-6.4	-23.9	-21.7	-27.8	10.5	0.66	-*
27	22.6	18.1	13.2	-9.4	34.3	2.7	0.17	-AG-BD
28	38.5	-4.4	-9.7	-3.4	-36.8	0.7	0.04	-*
29	29.4	2.6	-8.1	-20.2	-35.7	-18.1	-1.13	-AH-CD
30	35.5	15.9	-22.5	-22.9	-12.8	-2.5	-0.16	-*
31	31.3	6.1	13.3	-14.4	-2.7	22.9	1.43	-*
32	25.3	-6.0	-12.1	-25.4	-11.0	-8.3	-0.52	-AF-DE
	979.5	957.8	885.2	781.6	720.0	809.6	50.62	

Checks 957.8
885.2
781.6
720.0
809.6
50.60

A-2. Reverse Yates' Algorithm computation for thirty-two effects assuming all effects are zero except those for average, E, A, H, FH and F.

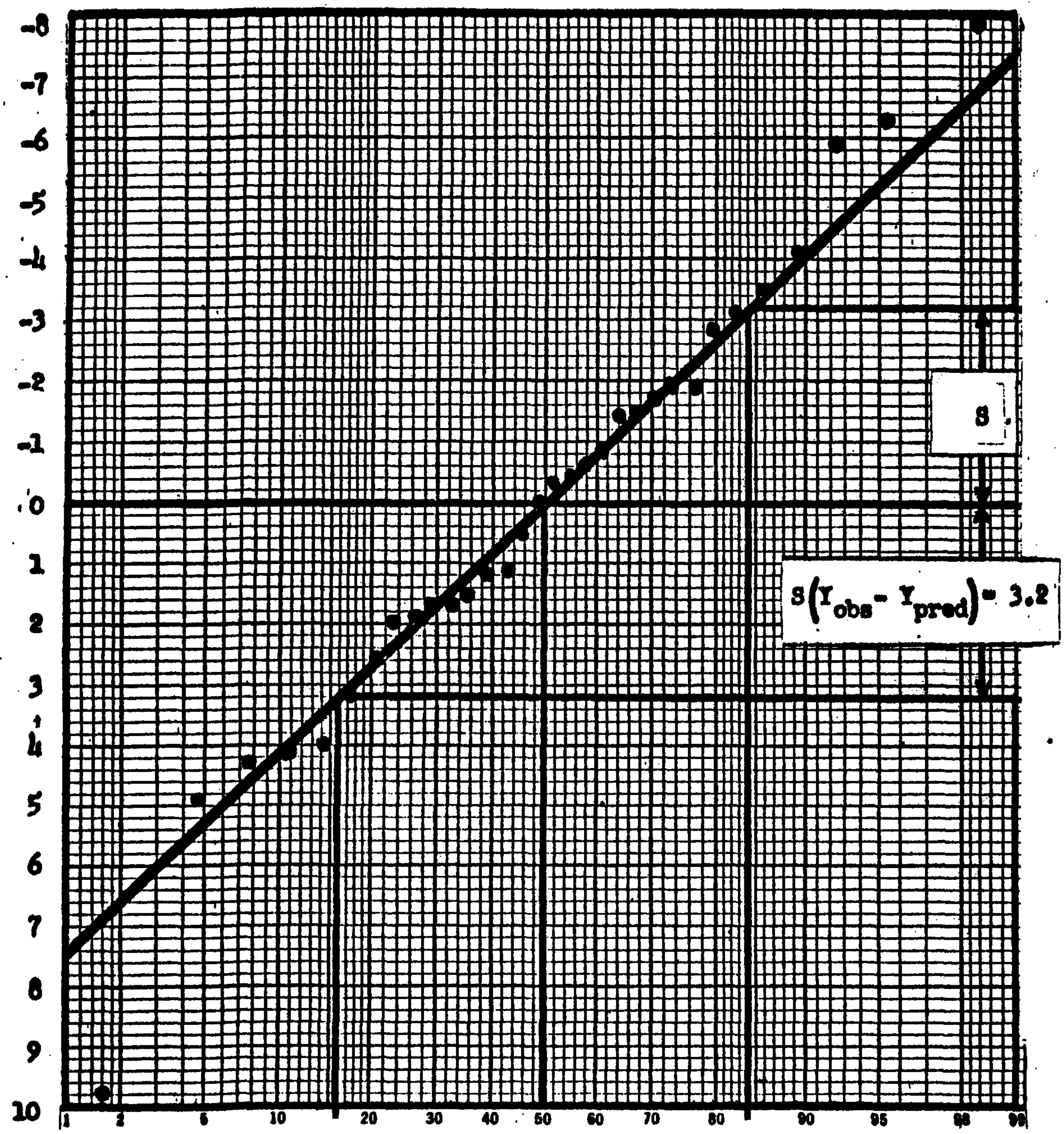
No.	Effects Measured	(5)	(5')	(6')	(7')	(8')	(9')	(10')	Predicted Response (10')/32
1	1	979.5	979.5	979.5	926.0	1,008.9	1,128.5	1,205.4	37.7
2	-	-21.7	0	53.5	-82.9	-119.6	-76.9	850.6	26.8
3	-	-19.1	0	0	0	-76.9	927.5	1,049.6	32.8
4	CE+FH	-53.5	-53.5	82.9	119.6	0	76.9	812.4	25.4
5	-	4.1	0	0	-76.9	950.1	972.7	800.4	25.0
6	-	-23.1	0	0	0	22.6	-76.9	1,061.6	33.2
7	-	-1.7	0	71.1	0	76.9	889.3	1,170.2	36.6
8	E	-82.9	-82.9	-48.5	0	0	76.9	885.8	27.7
9	-	43.5	0	-76.9	1,033.0	950.1	723.5	966.2	30.2
10	-	8.7	0	0	82.9	-22.6	-76.9	895.8	28.0
11	-	-5.9	0	0	0	-76.9	1,138.5	1,004.4	31.4
12	-	62.9	0	0	-22.6	0	76.9	1,051.6	32.9
13	-	-37.5	0	0	76.9	1,008.9	1,093.3	1,039.6	32.5
14	H	-71.1	-71.1	0	0	119.6	-76.9	1,016.4	31.8
15	F	-48.5	-48.5	0	0	76.9	962.7	1,215.4	38.0
16	-	-13.7	0	0	0	0	76.9	646.6	20.2
17	-	-18.3	0	979.5	1,033.0	843.1	889.3	1,051.6	32.9
18	-A	76.9	76.9	-53.5	82.9	119.6	-76.9	1,040.4	31.4
19	-	-30.5	0	0	0	-76.9	972.7	895.8	28.0
20	-	-5.3	0	-82.9	22.6	0	76.9	966.2	30.2
21	-	29.1	0	0	-76.9	1,115.9	972.5	646.6	20.2
22	-	23.1	0	0	0	-22.6	-76.9	1,215.4	38.0
23	-	13.7	0	-71.1	0	76.9	1,128.5	1,016.4	31.8
24	-	3.3	0	-48.5	0	0	76.9	1,039.6	32.5
25	-	-10.3	0	76.9	926.0	1,115.9	962.7	812.4	25.4
26	-	10.5	0	0	-82.9	22.6	-76.9	1,049.6	32.8
27	-	2.7	0	0	0	-76.9	1,093.3	850.6	26.6
28	-	0.7	0	0	-119.6	0	76.9	1,205.4	37.7
29	-	-18.1	0	0	76.9	843.1	1,138.5	885.8	27.7
30	-	-2.5	0	0	0	-119.6	-76.9	1,170.2	36.6
31	-	22.9	0	0	0	76.9	723.5	1,061.6	33.2
32	-	-8.3	0	0	0	0	76.9	800.4	25.0
			979.5 -48.5 931 x2 1,862	1,862 1,959 x2 3,918	3,918 x2 7,836	7,836 x2 15,672	15,672 x2 31,344	31,344	980.0 (979.5)

A-3. Comparison of observed and predicted responses.

No.	Observed Responses	Predicted Response	Obs. Resp.- Pred. Resp.	Ordered Series
1	33.6	37.7	-4.1	-9.8
2	28.0	26.6	1.4	-4.9
3	33.2	32.8	0.4	-4.3
4	23.9	25.4	-1.5	-4.1
5	23.3	25.0	-1.7	-4.0
6	30.6	33.2	-2.6	-3.2
7	40.0	36.6	3.4	-2.6
8	23.4	27.7	-4.3	-2.0
9	34.3	30.2	4.1	-1.9
10	28.6	28.0	0.6	-1.7
11	33.3	31.4	1.9	-1.7
12	40.8	32.9	7.9	-1.5
13	38.4	32.5	5.9	-1.2
14	29.8	31.8	-2.0	-1.1
15	38.0	38.0	0	-0.5
16	19.7	20.2	-0.5	0
17	28.0	32.9	-4.9	0.3
18	29.7	31.4	-1.7	0.4
19	26.8	28.0	-1.2	0.6
20	20.4	30.2	-9.8	0.8
21	23.0	20.2	2.8	1.4
22	41.1	38.0	3.1	1.5
23	33.7	31.8	1.9	1.7
24	29.3	32.5	-3.2	1.9
25	31.7	25.4	6.3	1.9
26	34.3	32.8	1.5	2.8
27	22.6	26.6	-4.0	3.1
28	38.5	37.7	0.8	3.4
29	29.4	27.7	1.7	4.1
30	35.5	36.6	-1.1	5.9
31	31.3	33.2	-1.9	6.3
32	25.3	25.0	0.3	7.9
	979.5	980.0	-44.5 +44.0 -0.5	

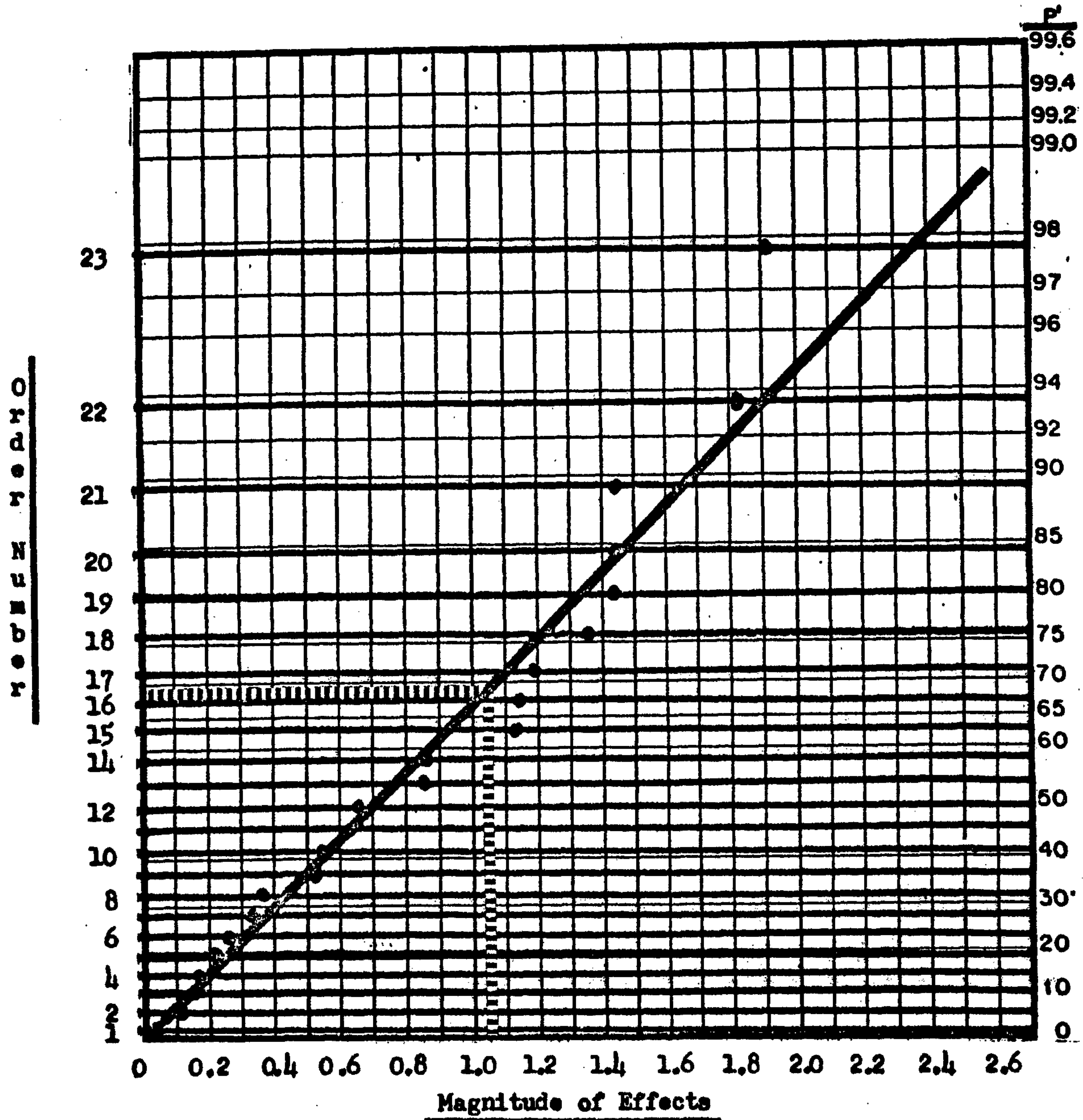
A - 4

Probability plot to obtain standard error of individual differences between observed and predicted responses



Probability Scale

A - 5 **Half-normal plot of twenty-three mean effects to obtain standard error of individual observed responses.**



$$S_Y = \frac{1.04}{\sqrt{\frac{1}{16}+\frac{1}{16}}} = 3.0$$